International Journal of Engineering, Science and Mathematics (UGC Approved)
Vol. 6 Issue 8, December 2017,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gate as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Multivariate GARCH models for stock indices volatility on Nairobi Securities Exchange

Ngassima Fanny Laure[1], Joseph K. Mung'atu[2], MbeleBidima Martin[3]

1 Pan African University Institute of Basic Sciences, Technology and Innovation (Pauisti)Nairobi, Kenya

2 Department of Mathematics (Finance) - Jomo Kenyatta University of Agriculture and Technology (Jkuat) Nairobi, Kenya

Abstract.

In the financial sector, volatility is one of the important aspects that need special attention as far as risk management is concerned. A multivariate GARCH model is presented together with its univariate specifications. This paper reviews the substantial literature on specifications, estimation, and evaluation of the MGARCH models. The quasi maximum likelihood technique is expanded to allow for estimation of GARCH-type models and is applied to the MGARCH models. Therefore, empirical results suggest that the best multivariate GARCH model is revealed to be the DCC model which dominates the others with respect to the likelihood values.

Keywords: Multivariate GARCH, Quasi maximum likelihood, Univariate GARCH

1. Introduction

Changes in volatility over time can be modelled using many approaches. The main characteristic of any financial asset is its return, which is typically considered to be a random variable.The asset's volatility that describes the spread of outcomes of this variable, plays the principal role in numerous financial applications. We often use it to estimate the value of market risk and we used it in this work for portfolio management. Then, the main purpose of this paper is to allow financial institutions not only to know the current value of the volatility of the managed assets, but also to be able to estimate their future values. However, an understanding of linkages and volatility transmission in stock market returns and correlation of such returns will help investors or fund managers better manage their investment portfolios. Indeed, with the crisis of confidence in risk management and requirements of regulators, there is a requirement for GARCH modeling to take explicitly into account multivariate issues. Nevertheless, employing a multivariate framework has always been a challenge concerning robust models and their estimation problems while considering the high number of parameters. Hence, the generalizations to multivariate series can be difficult to estimate and interpret.Another approach is to model volatility as an unobserved stochastic process which is subject of next results. A number of papers have documented the advantage of modelling stochastic volatility including Harvey et Al (1994) who used the Quasi Maximum Likelihood (QML) methods. Although there have been already some practical and successful applications of multivariate GARCH models, the theoretical literature on univariate GARCH models has developed significantly over the last few years.Yet the MGARCH models remain more difficult to estimate, although estimation is already an issue for the univariate GARCH models, it is believed that estimation is more of an issue for MGARCH models. Moreover, as a result of difficulties with parameter estimation, the computation of model comparison criteria becomes extensive and

demanding. Also, compared to the multitude alternative specifications in univariate GARCH models, only a handful of MGARCH model specifications have been studied; this may be among the multiple reasons why the MGARCH models have had fewer empirical applications.
Our interests in Multivariate GARCH models stem from their popularity in analysis of econometric and financial market data. Although, it has been shown byHol and Koopman (2002) that in some empirical studies Stochastic Volatility models make better forecasts than GARCH models do. In GARCH-type models, the conditional variance of returns is assumed to be a deterministic function of past returns. There are both economic and econometric reasons why multivariate volatility models are important. The knowledge of correlation structures is very important in many financial applications, such as asset pricing, optimal portfolio, risk management and asset allocation, so that multivariate volatility models are useful for making financial decisions. Two classes of models ARCH and Stochastic Volatility have emerged as the dominant approaches for modelling financial volatility. One of the main objectives of the study of time series is therefore, the forecasting of future realizations very often for economic reasons, namely to predict the evolution of a financial market.The method developed in this paper for estimation of the MGARCH model is Quasi maximum likelihood (QML). The model will be easy to estimate. It implies that volatility is modelled as a conditional variance.

The data used in this study is daily Equity Group and KCB Group Ltd stock prices data from 2010-2016. The paper is organized as follows. A theoretical survey of univariate GARCH models is presented in section 2, while section 3 collects theoretical survey of multivariate GARCH framework, containing the following models, VGARCH, Diagonal VGARCH, CCC and DCC. For each class of the model, a theoretical review, basic properties and estimation procedure are provided. Estimation results are presented in section 4 and section 5 concludes. Technical details are given in Appendix A.

2. Generalized ARCH models

In recent years, a variety of models which apparently forecast changes in stock market prices have been introduced, and have played an important role to help people forecast the future.
The ARCH class of model introduced by Engle (1982) and its generalization, GARCH models by Bollerslev (1986) are the most and widely used methodologies in modelling and forecasting volatility of financial time series. The literature of ARCH-type models is developed and we used it to model and forecast stock indices on NSE. In this chapter we studied different univariate and multivariate GARCH models. We also use the QML Estimation which is the common estimate method for this type of models.

2.1 GARCH (p, q) Model

Bollerslev (1986) introduced an extension of the ARCH model, with the following specifications. Let $\{e_t\}_{t\in\mathbb{Z}}$ a sequence of i.i.d random variables such that $e_t \sim N(0,1)$. Here, $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is said to be GARCH (p, q) process if:

$$r_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = \sigma_t e_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (2.1)$$

where p and q are the orders of the process, ω, α_i, and β_j are the parameters to be estimated, with $\omega > 0$, $\alpha_i \geq 0\left(for\ i = 1,\ldots,p\right)$ and $\beta_j \geq 0$ $\left(for\ j = 1,\ldots,q\right)$. These are the necessary conditions for the variance to be positive (CHOO,1999).

If we let $q = 0$, the process reduces to an ARCH (p) process and for $p = q = 0$, ε_t is simply a white noise. However, the short run dynamics of the resulting volatility process is determined by the size of the parameters α_i and β_j.

Large ARCH coefficients, α_i imply that volatility reacts significantly to markets movements, while large GARCH coefficients β_j indicate that shocks are persistent on the stocks market (Roberto Perrelli,2001).

We can also write the variance σ_t^2 of equation (2.1) in terms of the lag-operator L where $\left(\mathrm{L}\varepsilon_t = \varepsilon_{t-1}\right)$ we get:

$$\sigma_t^2 = \omega + \alpha\left(\mathrm{L}\right)\varepsilon_t^2 + \beta\left(\mathrm{L}\right)\sigma_t^2 \qquad (2.2)$$

Where

$$\alpha\left(\mathrm{L}\right) = \alpha_1\mathrm{L} + \alpha_2\mathrm{L}^2 + \ldots + \alpha_p\mathrm{L}^p \ and\ \beta\left(\mathrm{L}\right) = \beta_1\mathrm{L} + \beta_2\mathrm{L}^2 + \ldots + \beta_q\mathrm{L}^q \qquad (2.3)$$

Moreover, if

$$1 - \left(\alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_p x^p\right) = 0$$

that means if the roots of the characteristic equation lie outside the unit circle and the process $\left\{\varepsilon_t\right\}$ is stationary, then we can write the variance equation of equation (2.1) as

$$\sigma_t^2 = \frac{\omega}{1 - \beta\left(1\right)} + \frac{\alpha\left(\mathrm{L}\right)}{1 - \beta\left(\mathrm{L}\right)}\varepsilon_t^2 \qquad (2.4)$$

Now let $\tilde{\omega} = \frac{\omega}{1 - \beta\left(1\right)}$, and γ_i the coefficients of L^i in the expansion of $\frac{\alpha\left(\mathrm{L}\right)}{1 - \beta\left(\mathrm{L}\right)}$ then, we obtain the following transformation of equation (2.4)

$$\sigma_t^2 = \tilde{\omega} + \sum_{i=1}^{\infty}\gamma_i\varepsilon_{t-i}^2 \qquad (2.5)$$

We have demonstrated that the GARCH (p, q) can also be written as an ARCH$\left(\infty\right)$ process with a fractional structure of the coefficients. This clearly means that ε_t is also a martingale difference and the conditional variance of ε_t is given by

$$\sigma_t^2 = \frac{\omega}{1 - \sum_{i=1}^{p}\alpha_i - \sum_{j=1}^{q}\beta_j} \qquad (2.6)$$

with $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1$ to ensure the stationarity of the conditional variance.

GARCH models have been extended by many others authors in order to fill the gaps in the main GARCH model.

2.2 The symmetric GARCH (1, 1) model

In the GARCH (1, 1) model, the dynamics show up in the ACF of the squared returns and the ACF is like that of the ARMA (1, 1) process. If $\alpha+\beta$ is close to one then, the ACF will decay slowly indicating a relatively slowly changing conditional variance. In this model, the conditional variance is presented as a linear function of its own lags. It is a particular case of GARCH (p, q) where $p=q=1$. The basic univariate GARCH (1, 1) is given by

$$mean\ equation \quad r_t=\mu+\varepsilon_t \quad , \quad \varepsilon_t \sim N\left(0,\sigma_t^2\right) \tag{2.7}$$

$$variance\ equation \quad \sigma_t^2=\omega+\alpha_1\varepsilon_{t-1}^2+\beta\sigma_{t-1}^2 \tag{2.8}$$

where $\omega>0$, $\alpha_1\geq 0$, $\beta_1\geq 0$ and r_t is the return of the asset at time t, μ denotes the average return , σ_t^2 is the conditional variance and ε_t is the residual returns as defined in equation (2.4).
The size of parameters α and β determine the short-run dynamics of the volatility time series and if $\alpha_1+\beta_1=1$, then any shock will lead to a permanent change in all future values. Hence, shock to the conditional variance is "persistence".
The constraints $\alpha_1\geq 0$ and $\beta_1\geq 0$ in the GARCH (1, 1) are the conditions of positivity of the conditional variance (Ser-Huang,2005).
We can write the variance equation as a stochastic recurrence equation (SRE) by substituting the first equation (2.1) in equation (2.8), we obtain

$$\begin{aligned}\sigma_t^2&=\omega+\alpha_1\left(\sigma_{t-1}^2e_{t-1}^2\right)+\beta_1\sigma_{t-1}^2\\ &=\omega+\sigma_{t-1}^2\left(\alpha_1e_{t-1}^2+\beta_1\right)\end{aligned} \tag{2.9}$$

This can be written in the following form

$$\boldsymbol{X}_t=\boldsymbol{A}_t\boldsymbol{X}_{t-1}+\boldsymbol{B}_t \tag{2.10}$$

where $\{\alpha_t\}$ and $\{e_t\}, \forall t\in\mathbb{Z}$ are sequences of i.i.d random variables and with $\boldsymbol{X}_t=\sigma_t^2$, $\boldsymbol{X}_{t-1}=\sigma_{t-1}^2$, $\boldsymbol{A}_t=\alpha_1e_{t-1}^2+\beta_1$, and $\boldsymbol{B}_t=\omega$
These following conditions are sufficient to get a solution

$$\mathrm{E}\left(\ln^+|B_t|\right)<\infty \text{ and } \mathrm{E}\left(\ln|A_t|\right)<\infty \tag{2.11}$$

and the meaning of $\mathrm{E}\left(\ln^+|B_t|\right)$ is $\max\left\{0,\left(\ln|B_t|\right)\right\}$.

By iteration $n\ times$ we get from equation (2.13) the following expression

$$\begin{aligned}X_t&=A_t\left(A_{t-1}X_{t-2}+B_{t-1}\right)+B_t\\ &=B_t+\sum_{i=1}^{n}B_{t-i}\prod_{j=0}^{n-1}A_{t-j}+X_{t-k-1}\prod_{i=0}^{n}A_{t-i}\end{aligned}$$

Conditions given in (2.11) ensure that the middle term on the right hand side converges absolutely and the last term disappears as shown below

$$\frac{1}{n+1}\sum_{i=0}^{n}\ln|A_{t-i}|\rightarrow\mathrm{E}\left(\ln|A_t|\right)<0$$

and by the strong law of large numbers, this yields

$$\prod_{i=0}^{n}\left|A_{t-i}\right| = \exp\left(\sum_{i=0}^{k}\ln\left|A_{t-i}\right|\right) \to 0$$

Hence, the unique solution of equation (2.10) is given by

$$X_t = B_t + \sum_{i=1}^{\infty} B_{t-i}\prod_{j=0}^{i-1} A_{t-j} \qquad (2.12)$$

In this case, the sum $\sum_{i=1}^{\infty} B_{t-i}$ also converges absolutely almost surely. Then the general solution of equation (2.9) becomes

$$\sigma_t^2 = \omega\left(1 + \sum_{i=1}^{\infty}\prod_{j=1}^{i}\alpha_1 e_{t-1}^2 + \beta_1\right) \qquad (2.13)$$

Then, the solution of the GARCH (1, 1) defining equations is given by

$$\varepsilon_t = e_t\sqrt{\omega\left(1 + \sum_{i=1}^{\infty}\prod_{j=1}^{i}\alpha_1 e_{t-1}^2 + \beta_1\right)} \qquad (2.14)$$

2.3 The Asymmetric EGARCH model

This model is based on the logarithmic expression of the conditional variability. We use this model to capture the asymmetric responses of the time varying volatility and returns at the same time, whenever the parameter values are negative, the model ensures that the conditional variance is always positive (Suliman and Winker,2012), this means that there is no need for parameter restrictions to impose non negativity. The model was developed by Nelson (1991) and hence, the following equation

$$\ln\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i\frac{\left|\varepsilon_{t-i}\right| + \delta_i\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q}\beta_j\ln\sigma_{t-j}^2 \qquad (2.15)$$

where δ is the asymmetric response parameter.

The EGARCH (p, q) conditional variance model includes q past log conditional variances that compose the GARCH component polynomial.

In most empirical cases, δ is expected to be negative so that a “negative shock” increases “future volatility”, while a positive shock eases the effect on future volatility (Harvey, 2013). Therefore, for an EGARCH (1, 1) model where $p = q = 1$given by

$$\ln\sigma_t^2 = \omega + \beta_1\ln\sigma_{t-1}^2 + \alpha_1\left\{\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \sqrt{\frac{\pi}{2}}\right\} - \delta\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \qquad (2.16)$$

The left hand side is the log of the conditional variance. The coefficient δ is known as the asymmetry or leverage term. The presence of leverage effects can be tested by the hypothesis that $\delta < 0$, the impact is symmetric if $\delta \neq 0$and $\pi = 22/7$.

3. Multivariate GARCH Models

Nowadays, globalization has resulted in higher international economics integration, investors and also financial institutions are interested in knowing financial markets integration and how financial volatilities together move over time across several markets or assets.Empirical results show that working with separate univariate models is much less relevant than multivariate modelling framework.The most common application of these class of models is to estimate the volatility effects among different markets or assets. In MGARCH models, covariance matrix need by definition to be positive definite, therefore imposing positive definiteness is one of the features that needs to be taken into account in its specifications. One possibility is to derive conditions under which the conditional variance matrices implied by the model are positive definite, but this is often not feasible in practice. In this case, an alternative is to formulate the model in a way that positive definiteness is implied by the structure (in addition to some simple constraints).

3.1 VGARCH Model

Bollerslev, Engle and Wooldridge (1988) proposed a VGARCH model which is a straightforward generalization of the univariate GARCH model.

Every conditional variance and covariance is function of all lagged conditional variance and covariance, as well as lagged squared returns and cross products of returns. The VGARCH is defined as follow:

Definition 3.1

A VGARCH (p, q) process is a martingale difference sequence X_t, relative to a given filtration F_t, whose conditional covariance matrix $H_t = \text{cov}(X_t / F_{t-1})$ satisfy, $\forall t \in$ □

$$\text{Vech}(H_t) = \omega + \sum_{i=1}^{p} A_i\, vech\left(X_{t-i}X'_{t-i}\right) + \sum_{i=1}^{q} B_i\, vech\left(H_{t-i}\right) \qquad (3.1)$$

where $vech(.)$ is the operator that stocks the lower triangular portion of a symmetric square $k \times k$ matrix into a $(k(k+1)/2) - \text{dim}ensional$ vector. ω is an $(k(k+1)/2)$ dimensional vector, A_i and B_i are square parameter matrices of order $(k(k+1)/2)$

For a purpose of explanation, let's consider a bivariate VGARCH (1, 1) model with $k = 2$ and we denote $\sigma_t^2 = vech(H_t)$, the equation (2.30) becomes

$$\sigma_t^2 = \begin{pmatrix} \sigma_{11,t}^2 \\ \sigma_{12,t}^2 \\ \sigma_{22,t}^2 \end{pmatrix} = \begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \omega_{3,t} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{bmatrix} X_{1t-1}^2 \\ X_{1t-1}X_{2t-1} \\ X_{2t-1}^2 \end{bmatrix} + \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{bmatrix} \sigma_{11t-1}^2 \\ \sigma_{12t-1}^2 \\ \sigma_{22t-1}^2 \end{bmatrix}$$

We can notice that, from this we immediately see equivalency VEC and VECH representation.

In VEC representation all the covariance equations appear twice, because there is an equation for $\sigma_{ij,t}^2$ as well as for $\sigma_{ji,t}^2$. This is because all the off-diagonal terms appear twice within each equation. (i.e. both of the terms $\sigma_{ij,t-1}^2$ and $\sigma_{ji,t-1}^2$ appear in each equation). We can then remove this redundant terms without affecting the model doing so, dimensions of matrices A_i and B_i become $\left(k\left(k+1\right)/2\right)$ instead of k^2.

This model is general and flexible, and the coefficients are also directly interpretable, but it has some drawbacks in applications, the higher number of parameters which equals $\left(p+q\right)\left(k\left(k+1\right)/2\right)^2+k+\left(k+1\right)/2$, however the model will be practicable in practice in our study since we use the bivariate case. Another disadvantage is, there exists only sufficient conditions on the parameters to ensure that conditional variance matrices H_t are positive definite almost surely $\forall t$.

The restrictions of the model are introduced by Bollerslev, Engle and Wooldridge (1988) such that, each component of the covariance matrix H_t depends only on its own past and past values of $X_t X_t^{'}$ as in equation (3.1), that means in the diagonal representation, it is assumed that the matrices A_i and B_i are diagonal, we call it a diagonal VECH model.

3.2 The Diagonal VGARCH model

This so called DVGARCH model will reduce the number of parameters to $\left(p+q+1\right)\left(k\left(k+1\right)/2\right)$ and therefore it is still possible to obtain conditions for positive definiteness of H_t $\forall t$.

To illustrate the bivariate case, the DVGARCH model is simply:

Letting $h_t=\sigma_t^2$,

$$\sigma_t^2=\begin{pmatrix}\sigma_{11,t}^2\\ \sigma_{12,t}^2\\ \sigma_{22,t}^2\end{pmatrix}=\begin{pmatrix}\omega_{1,t}\\ \omega_{2,t}\\ \omega_{3,t}\end{pmatrix}+\begin{pmatrix}a_{11}&0&0\\ 0&a_{22}&0\\ 0&0&a_{33}\end{pmatrix}\begin{bmatrix}X_{1,t-1}^2\\ X_{1,t-1}X_{2,t-1}\\ X_{2,t-1}^2\end{bmatrix}+\begin{pmatrix}b_{11}&0&0\\ 0&b_{22}&0\\ 0&0&b_{33}\end{pmatrix}\begin{bmatrix}\sigma_{11,t-1}^2\\ \sigma_{12,t-1}^2\\ \sigma_{22,t-1}^2\end{bmatrix}$$

We have

$$\sigma_{11,t}^2=\omega_{1,t}+a_{11}X_{1,t-1}^2+b_{11}\sigma_{11,t-1}^2$$

$$\sigma_{12,t}^2=\omega_{2,t}+a_{22}X_{1,t-1}X_{2,t-1}+b_{22}\sigma_{12,t-1}^2$$

$$\sigma_{22,t}^2=\omega_{3,t}+a_{33}X_{2,t-1}^2+b_{33}\sigma_{22,t-1}^2$$

In the bivariate model illustrated here, there are three free parameters in each of the A_1 and B_1 matrices and nine parameters (including constants). In the general $k-$variate DVGARCH model there are $\left(k\left(k+1\right)/2\right)$ free parameters in each matrix. However, the DVGARCH representation seems to be too restrictive since no interaction is allowed between the different conditional variances and covariance.

In order to derive a sufficient condition for the DVGARCH for H_t to be positive definite , we write the known DVGARCH model in a matrix representation yields

$$H_t=\tilde{W}+\tilde{A}\odot X_{t-1}X_{t-1}^{'}+\tilde{B}\odot H_{t-1} \qquad (3.2)$$

where $\odot$ denotes the element-by-element product of the two matrices. $\tilde{W}$, $\tilde{A}$, and $\tilde{B}$ are all $k \times k$ parameter matrices. Using Cholesky decomposition of the parameter matrices and from the properties of a Hadamard product yields

$$H_t = \tilde{W}\tilde{W}' + \tilde{A}\tilde{A}' \odot X_{t-1}X_{t-1}' + \tilde{B}\tilde{B}' \odot H_{t-1} \qquad (3.3)$$

where $\tilde{W}\tilde{W}'$, $\tilde{A}\tilde{A}'$, and $\tilde{B}\tilde{B}'$ are all positive semi-definite and therefore, H_t is positive definite $\forall t$, since the initial covariance matrix H_0 is also positive definite.

By writing the parameters matrices in the form of Cholesky decomposition, the positive semi-definiteness is guaranteed in estimation without imposing any further restrictions.

By definition we have the operator L as mentioned previously in the univariate case, where $L^i X_t = X_{t-i}$ and by convention, $A(L) = A_1 L + A_2 L^2 + \ldots + A_p L^p$ and $B(L) = B_1 L + B_2 L^2 + \ldots + B_q L^q$. Now let $\boldsymbol{z}_t$ our $k-$dimensionali.i.d vector process with mean zero and unit variance, knowing that $\boldsymbol{z}_t$ is independent of F_{t-1}, it follows that $\mathrm{cov}(\boldsymbol{z}_t / F_{t-1}) = \mathrm{cov}(\boldsymbol{z}_t) = I_n$. There exists a VGARCH process X_t such that $X_t = \sqrt{H_t}\boldsymbol{z}_t$ where $H_t = \mathrm{cov}(X_t / F_{t-1})$ and $F_t = \sigma(X_t, X_{t-1}, \ldots.)$.

Assuming that X_t is doubly infinite sequence, yields to the following equation for conditional covariance matrix, by rewriting equation (3.1) as

$$vech(H_t) = \sum_{i=1}^{\infty} B(L)^{i-1}\left[\omega + A(L)vech(X_t X_t')\right] \qquad (3.4)$$

3.3 CCC Model

Introduced for the first time by Bollerslev, the conditional correlation matrix in this class of models is time invariant. We then choose a GARCH-type model for each conditional variance and we model the conditional correlation matrix, based on the conditional variances.

Since the conditional correlation matrix is time invariant, the conditional covariances are therefore proportional to the product of the corresponding conditional standard deviations. Hence,

Definition 3.2

The CCC (p, q) process is a martingale difference sequence X_t, relative to a given filtration F_t, whose conditional covariance matrix $H_t = \mathrm{cov}(X_t / F_{t-1})$ satisfy

$$H_t = D_t R D_t = \rho_{ij}(\sigma_{iit}\sigma_{jjt}) \qquad (3.5)$$

where $$D_t = \mathrm{diag}(\sigma_{11t}, \ldots, \sigma_{kkt}) \qquad (3.6)$$

and $$R = (\rho_{ij}) \qquad (3.7)$$

is a symmetric positive definite matrix with $\rho_{ii} = 1,\ \forall i$ then off diagonal elements of the conditional covariance matrix are defined as $[H_t]_{ij} = \sigma_{it}\sigma_{jt}\rho_{ij}$ for $i \neq j,\ 1 \leq i, j \leq k$. σ_{iit}^2 is defined as univariate GARCH (p, q) model

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} A_i X_{t-i}^2 + \sum_{i=1}^{q} B_i \sigma_{t-i}^2 \qquad (3.8)$$

where ω is $k \times 1$ vector, A_i and B_i are diagonal $k \times k$ matrices. See Francq and Zakoian (2010) for more details.

3.4 DCC Model

A generalization of the CCC model was proposed by Engle (2002), the so-called DCC is a new class of multivariate models which conditional correlation matrix is time-dependent. These models are flexible like the previous univariate GARCH and parsimonious parametric models for the correlations.

Definition 3.3

The DCC process is a martingale difference sequence X_t, relative to a given filtration F_t, whose conditional covariance matrix $H_t = \text{cov}(X_t / F_{t-1})$ satisfy

$$H_t = D_t R_t D_t \qquad (3.9)$$

where

$$D_t = \text{diag}(\sigma_{1t}, \ldots, \sigma_{kt}) \qquad (3.10)$$

and R_t is $k \times k$ time varying correlation matrix of X_t, σ_{it}^2 is defined as univariate GARCH (p, q) model.

$$\sigma_{it}^2 = \omega_i + \sum_{j=1}^{p_i} \theta_{ij} X_{t-j}^2 + \sum_{j=1}^{q_i} \phi_{ij} \sigma_{t-j}^2$$

where ω_i, θ_{ij}, and ϕ_{ij} are non-negative parameters for $i = 1, \ldots, k$, with the usual GARCH restriction for non-negativity and stationary being imposed, such as non-negativity of variances and

$$\sum_{j=1}^{p_i} \theta_{ij} + \sum_{j=1}^{q_i} \phi_{ij} < 1.$$

In bivariate case, the number of parameters to be estimated equals $(k+1)(k+4)/2$. Note that H_t, being a covariance matrix has to be positive definite, D_t is positive definite since all the diagonal elements are positive, this ensure R_t to be positive definite. Also, all the elements in the correlation matrix R_t have to be equal or less than one by definition; See Engle (2002) for more details.

3.5 Model Estimation

Estimation of MGARCH models is troublesome, since the number of parameters may be large enough also for relative small vector dimension k.

Statistical properties of multivariate GARCH models are only known for development of statistical estimation, it would be desirable to have conditions for strict stationarity and ergodicity of multivariate GARCH processes as well as conditions for consistency and asymptotic normality of QMLE.

Comte and Lieberman (2003) establish asymptotic normality of the QMLE, they provide condition for strong consistency and asymptotic normality of the QMLE $\hat{\theta}$

The method of estimation of parameters of R_t in the DCC model produces consistent but not efficient estimators, In order to estimate the parameters of H_t we use the following log-likelihood function L

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log(|H_t|) + X_t^{'} H_t^{-1} X_t\right)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log(|D_t R_t D_t|) + X_t^{'} D_t^{-1} R_t^{-1} D_t^{-1} X_t\right)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + X_t^{'} D_t^{-1} R_t^{-1} D_t^{-1} X_t\right)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log(|D_t|) + X_t^{'} D_t^{-1} D_t^{-1} X_t - \boldsymbol{z}_t^{'}\boldsymbol{z}_t + \log|R_t| + \boldsymbol{z}_t^{'} R_t^{-1} \boldsymbol{z}_t\right)$$

$$= L_v(\theta) + L_c(\theta,\phi)$$

Thus, the log-likelihood is composed of two parts. The maximization of this function will be done in two steps as proposed by Engle (2002). Note that, the estimate of the CCC model requires only the first step. The first part of the likelihood function depends on the parameters of the volatility of each stock market estimated from the univariate GARCH models.

$$L_v(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|D_t|^2 + X_t^{'} D_t^{-2} X_t\right)$$

It is the sum of the log-likelihoods of the individual GARCH models.

$$L_v(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{k}\left(\log(2\pi) + \log(\sigma_{it}^2) + \frac{X_{it}^2}{\sigma_{it}^2}\right)$$

The second part of the likelihood function depends on the conditional correlation parameters, knowing the coefficients of the volatilities obtained during the first step. In this second phase, the standardized residuals are used to estimate the parameters of the dynamics of the correlations.

$$L_c(\theta,\phi) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|R_t| + \boldsymbol{z}_t^{'} R_t^{-1} \boldsymbol{z}_t - \boldsymbol{z}_t^{'}\boldsymbol{z}_t\right).$$

4. Empirical Results

4.1 The data

The data sequences are generated by the same source. Daily closing prices of NSE Equity and KCB shares data over a period of 7 years extending from 01/01/2010 to 31/12/2016 with 1756 observations were used. The Equity and KCB shares are the most traded and most profitable companies trading in NSE market. They track the daily performance of the most capitalized companies in the sector of Banking among the eight (08) segments listed on the NSE.In order to describe the behaviour of NSE return series, we drawn descriptive statistics table for the returns. The data are in log-difference form.

The summary of the descriptive statistics for the NSE returns series are shown in table 1. As it is expected for a time series of returns the mean is close to zero. The return series are both negatively skewed, an indication that the NSE data used have symmetric returns.

Table1:Summary statistics of NSE return series

Statistics	Equity Group	KCB Group Ltd
Observations	1757	1757
Max	0.0946	0.0878
Min	-0.1022	-0.1121
Mean	0.00043	0.00022
Variance	0.00037	0.00031
SD	0.0192	0.0176
Skewness	-0.093	-0.402
Kurtosis	7.3388	7.541
P-normal	< 5%	< 5%
correlation coef	0.2409	
corr coef for squared returns	0.2803	

The kurtosis is greater than three for the normal distribution, this indicates that the underlying distribution of the returns are leptokurtic or heavy tailed.

Table 2: *Univariate ARCH results. Log-likelihood value*

Model	Number of parameters	Equity	KCB
Observations		1756	1756
GARCH	3	4548.656	4696.95
EGARCH	4	4670.046	4821.672

The series fail the Kolmogorov normality test statistic which rejects normality at the 1% confidence level in both cases; that means they have positive excess kurtosis which confirms that the returns are effectively leptokurtic or heavy tailed. They are highly correlated, with the correlation coefficient equal to 24%, while the corresponding value for squared returns is 28%. This suggests that estimating a joint model may yield interesting information on the relationship between the stock index data.

4.2 ARCH estimation

The univariate ARCH results are reported in Table 2. Several ARCH models were estimated with data, we proved that the data returns are stationary with high volatility. The presence of an ARCH effect has also been proved using the test for conditional heteroscedasticity and the Ljung-Box test confirm the presence of ARCH effect in the residuals returns. In each model, conditional normality was assumed and we used the BIC and AIC tests to select the appropriate GARCH model which better fit the data. In all cases, the EGARCH model outperforms the GARCH model selected in log-likelihood values with the largest difference in log-likelihood values 121 and 124 for both Equity and KCB stock index respectively. The coefficients of the univariate model are all significant.

4.3 Multivariate GARCH models

Results from the estimation of multivariate GARCH models of the data are presented in Table 3 for bivariate data set, DCC model has the best

Table 3:*Multivariate GARCH results: Bivariate Log-likelihood values*

Model	Number of parameters	Stock indices
DVGARCH	9	9274.12
VGARCH	21	8877.301
CCC	9	9284.197
DCC	9	9,502

performance with the highest log-likelihood value among the GARCH models, and its restricted version; the CCC and diagonal VGARCH perform significantly worse and are almost the same, but it is up to the user to select the model that he wants to use in portfolio management.A probable explanation for that is that the optimization might have converged to local minima. The difference in likelihood values is therefore indicative of which model fits better. Non-linear optimization of higher parameters model is always difficult, and highlights one of the problems with the MGARCH models.

5. Conclusion

In summary, we have observed a number of apparent results and most of the models considered above are non-nested and model comparison is difficult. Therefore, the results reported in this paper support several conclusions, present theoretical and empirical modeling with multivariate GARCH models and highlighted their features. Although researchers have built many multivariate models, we still face the problems of curse of dimension due to the number of parameters and the restrictions on the parameters to ensure the positive definiteness of the covariance matrix. There exist several types of multivariate GARCH models and we surveyed their basic construction. We considered the VGARCH, diagonal VGARCH, CCC, and DCC models and we used multistep maximum likelihood estimation procedures to estimate the models. One of the main findings is that conditional correlations exhibit significant changes over time so, we concluded that despite the impact of globalization, there still exist opportunities to maximize portfolio returns through diversification. Our comparison of the models shows that the best model is DCC, because it dominates in log-likelihood value. The greatest challenge remains to compare these results to the discrete multivariate stochastic volatility.

Appendix A.

We analyzed daily data on returns of Equity and KCB stocks. We specified one ARCH term and one GARCH term for the conditional variance equation of each company.

Table A1. Estimated coefficients of the diagonal VGARCH model for the stock indices

	C1	C2	ω_{11}	ω_{21}	ω_{22}
Estimate	0.00012	0.00019	0.000098	0.000027	0.000091
Std. Error	0.00041	0.00038	0.000017	0.000014	0.000015
	a11	a21	a22	b11	b21
Estimate	0.204	0.055	0.208	0.534	0.523
Std. Error	0.035	0.026	0.033	0.066	0.228
	b22				
Estimate	0.499				
Std. Error	0.063				

Table A2. Estimated coefficient of the CCC model for the Stock indices

	C Estimates		*ARCH Estimates*		*GARCH Estimates*	
	C1	C2	[,1]	[,2]	[,1]	[,2]
Estimate	0.0001	0.000087	0.215	0.213	0.519	0.506
Std. Error	0.000016	0.000014	0.353	0.034	0.063	0.062

Table A3. Estimated coefficients of the DCC model for Stock indices.

	ARCH Estimates		*GARCH Estimates*		*C Estimates*	
	[,1]	[,2]	[,1]	[,2]	C1	C2
Estimate	0.235	0.275	0.235	0.466	-0.00019	0.00066
Std. Error	0.05	0.053	0.524	0.08	0.00034	0.00031
	dcc a	dcc b	Corr.			
Estimate	0.273	0.496	0.168			
Std. Error	0.024	0.586	0.027			

FigureA1.Prediction of Conditional Covariance between the two companies

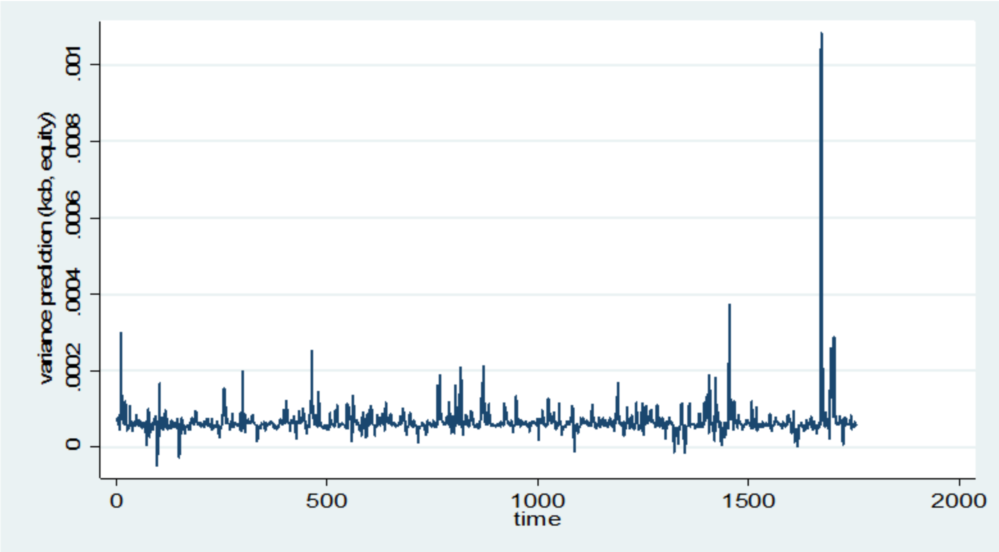
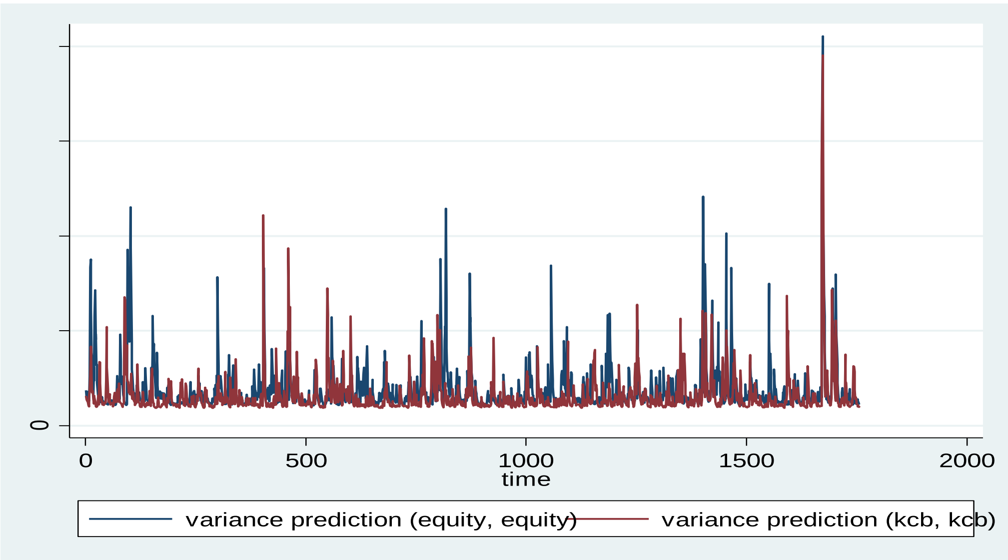


Figure A2. One-step prediction of volatility over the sample



References

Akaike, H. (1974). A new look at the Statistical model identification . *IEEE Transactions on Automatic Control, 19(6)*, 716-1974.

AL-Najjar, D. (2016). Modelling and Estimation of Volatility Using ARCH/GARCH Models in Jordan's Stock Market. *Asian Journal o Finance and Accounting, 8*, 152-167.

Bauwens, L., Laurent, S., & Rombouts, J. (2006). Multivariate GARCH models: A survey. *Journal of Applied Eonometrics, 21*, 79-109.

Berkes, I., Horvath, L., & Kokoszka, P. S. (2003). GARCH Processes: Structureand estimation. *Bernoulli, 9*, 201-227.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroscedasticity . *Journal of Econometrics, 31*, 307-327.

Bollerslev, T., & Wooldridge, J. (1992). Quasi-Maximum likelihood estimation and Inference in dynamic models with time-varying covariances. *Econometrics Reviews, 11*, 143-172.

Bollerslev, T., Engle, R., & Wooldridge, J. (1988). A capital Asset Pricing model with time-varying covariances. *The Journal of Political Economy, 96*, 116-131.

Chang, C.-Y., Xi-Yuan Q., & Sheng-Yuan J. (2011). Bayesian estimation and Comparison of MGARCH and MSV models via WinBUGS. *Journal of Statistical Computation and Simulation, 82:1*, 19-39.

Chong, C. W., Muhammad Ahmad, & Mat, Y. (1999). Performance of GARCH Models in Forecasting Stock Market Volatility. *Journal of Forecasting, 18*, 333-343.

Comte, F., & Lieberman, O. (2003). Asymptotic theory for multivariate GARCH processes. *Journal of Multivariate Analysis, 84*, 61-84.

Dickey D. and Fuller, W. (1981). Distribution of the estimators for autoregressive time series with a unit root. *Econometrica, 49*, 1057-1072.

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with estimates of the variance of United Kingdom Inflation. *Eonometrica, 50*, 122-150.

Engle, R. F. (1982). The use of ARCH/GARCH Models in Applied Econometrics. *Journal of Economic Perspectives, 15(4)*, 157-168.

Engle, R. F. (2002). Dynamic Conditional Correlation. A Simple Class of Multivariate Generalized Autoregressive conditional Heteroscedasticity models. *Journal of Business and Economic statistics, 20*, 339-350.

Engle, R. F., & Kroner, K. (1995). Multivariate Simultaneous Generalized ARCH. *Econometric Theory, 11*, 122-150.

Engle, R. F., & Sheppard, K. (2001). Theoretical and Empirical properties of Dynamic Conditional Correlation Multivariate GARCH. *NBER Working Papers 8554, National Bureau of Economic Research, Inc.*

Francq, C., & Zakoian, J. (2004). Maximum Likelihood estimation of pure GARCH-ARMA processes. *Bernoulli, 10(4)*, 605-637.

Francq, C., & Zakoian, J.-M. (2010). QML Estimation of a class of multivariate GARCH models without moment conditions on the observed process. *MPRA Paper No. 20779, Unpublished*.

Hamilton, J. D. (1994). Time series Analysis. *Princeton University Press, Princeton New Jersey*.

Harvey, A., & Genaro Sucarrat. (2013). EGARCH models with fat tails, skewness and leverage. *Computational Statistics and Data Analysis*.

Harvey, Esther Ruiz, & Neil Shephard. (1994). Multivariate Stochastic variance Models. *OxFord University Press, The Review of Economic Studies, 61*, 247-264.

Koopman, S. J., & Eugenie Hol U. (2002). The Stochastic Volatility in Mean Model. Empirical Evidence from International Stock Markets. *Journal of Applied econometrics*, 667-689.

Nelson, D. B. (1991). Conditional Heteroscedasticity in assets returns: a new approach. *Econometrica, 59*, 347-370.

Perrelli, T. R. (2001). Introduction to ARCH and GARCH models . *University of Illinois. Department of Economics. Econ 472*.

Poon, S.-H. (2005). A practical Guide to Forecasting Financia Market Volatility. *John Wiley and Sons, Ltd*.

Sheppard, K. (revised 25 Oct. 2003). Multi-step estimation of Multivariate GARCH models. *Proceedings of the International ICSC symposium: Advanced Computing in Financial Markets, University of California at San Diego*.

Suliman, Z. S., & Winker, P. (2012). Modelling Stock Market Volatility Using Univariate GARCH models: Evidence from Sudan and Egypt . *International Journal of Economics and Finance, 4(8)*, 161-176.

Taylor, & Francis. (2016). Robust Parametric Tests of Constant Conditional Correlation in MGARCH model. *Econometric Reviews*.

Taylor, S. J. (2005). Asset Price Dynamics, Volatility and Prediction. *Princeton University Press*.

Tsay, R. (2010). Analysis of Financial Times Series. *John Wiley and Sons, Inc., New York, United States of America, 3rd edition*.

TSE, Y. K., & TSUI, K. (2002). A multivariate Generalized autoregressive conditional heteroscedasticity model with time-varying correlation. *Journal of Business and Economic statistics, 20*, 351-362.